

**Logical Reasoning** (Pages 37–42)

The statement *If it is raining outside, then I will wear my raincoat* is called a conditional statement. All **conditional statements** can be written in the form *If A, then B*. Statements of this form are known as **if-then statements**. *A*, the portion of the statement immediately following *if*, is called the **hypothesis**. *B*, the portion of the statement immediately following *then*, is called the **conclusion**.

The process of using definitions, rules, properties, or facts as a means of validating conditional statements is **deductive reasoning**. If a true conditional exists, with a known true hypothesis, then deductive reasoning permits the reader to acknowledge that the conclusion is true for the scenario. A counterexample can be used to show that a conditional is not correct. A **counterexample** is a specific situation in which a statement is false. Only one counterexample is necessary to show that a statement is incorrect.

**Examples**

- a. **Identify the hypothesis and the conclusion.**

If  $3a + 12 = 24$ , then  $a = 4$ .  
 Hypothesis:  $3a + 12 = 24$   
 Conclusion:  $a = 4$

- b. **Write the conditional in if-then form.**

*I will attend the school play on Friday.*  
 Hypothesis: *It is Friday*  
 Conclusion: *I will attend the school play*  
 If *it is Friday*, then *I will attend the school play*.

**Try These Together**

*Identify the hypothesis and the conclusion. Write in if-then form.*

- I will earn an A for a score of 90% or higher.
- Tom will play inside when the weather is bad.

**Practice**

Use deductive reasoning to verify whether each conditional is *true* or *false*. If it is false, provide a counterexample.

- If there is a rainbow, then it must have rained while the Sun was shining.
- If the flowers are wet, then it rained.

5. **Standardized Test Practice** Which numbers are counterexamples for the conditional statement.

*If  $x \cdot y = 60$ , then  $x$  and  $y$  are positive numbers.*

- A  $x = 10, y = 6$       B  $x = 3, y = 20$       C  $x = -2, y = -30$       D  $x = 1, y = 60$

## 2-3 Multiplying Rational Numbers (Pages 79–83)

The product of two numbers having the *same sign* is positive. The product of two numbers having *different signs* is negative. It is also useful to note that multiplying a number or expression by  $-1$  results in the opposite of the number or expression. This is called the **multiplicative property of  $-1$** .

### Examples

a. Evaluate  $-3x^2$  for  $x = -\frac{2}{3}$ .

$$\begin{aligned} -3x^2 &= -3\left(-\frac{2}{3}\right)^2 && \text{Replace } x \text{ with } -\frac{2}{3}. \\ &= -3\left(\frac{4}{9}\right) && \left(-\frac{2}{3}\right)^2 = -\frac{2}{3} \cdot \left(-\frac{2}{3}\right) \text{ or } \frac{4}{9} \\ &= -\frac{12}{9} && \text{Divide out common factors.} \\ &= -\frac{4}{3} \text{ or } -1\frac{1}{3} && \text{Multiply. The signs are different,} \\ &&& \text{so the product is negative.} \end{aligned}$$

b. Simplify  $(-1)(2x)(-3y) + (4x)(-5y)$

$$\begin{aligned} &(-1)(2x)(-3y) + (4x)(-5y) \\ &= 2x(-1)(-3y) + (4x)(-5y) && \text{Commutative Property} \\ &= 2x(3y) + (-20xy) && \text{Multiply.} \\ &= 6xy + (-20xy) && \text{Multiply.} \\ &= -14xy && \text{Combine like terms.} \end{aligned}$$

### Practice

Find each product.

- |   |  |   |  |
|---|--|---|--|
| 1. $(-2)(3)(-5)$  | 2. $5.26(-0.011)$  | 3. $-10.01(-10.11)$   | 4. $2\left(\frac{3}{5}\right)\left(-\frac{5}{7}\right)$    |
| 5. $\left(-\frac{8}{11}\right)\left(\frac{9}{10}\right)$    | 6. $\left(-\frac{7}{10}\right)\left(-\frac{13}{21}\right)$ | 7. $\left(-\frac{8}{13}\right)(0)\left(-\frac{4}{5}\right)$ | 8. $3\left(\frac{4}{9}\right)(-4)\left(\frac{6}{7}\right)$ |
| 9. $\left(-\frac{2}{5}\right)(-4)\left(-\frac{3}{8}\right)$ | 10. $5\left(\frac{3}{4}\right)(-4)(-2)$                    | 11. $8(-0.25)(-3)$  | 12. $\frac{2}{7}(-21)(13)\left(\frac{1}{14}\right)$        |

Evaluate each expression if  $r = -\frac{1}{8}$ ,  $s = \frac{4}{5}$ ,  $t = -2\frac{9}{10}$ , and  $w = -1\frac{2}{9}$ .

- |           |           |              |                                    |
|-----------|-----------|--------------|------------------------------------|
| 13. $4rs$ | 14. $2tw$ | 15. $rt - s$ | 16. $s^2\left(-\frac{1}{8}\right)$ |
|-----------|-----------|--------------|------------------------------------|

Simplify.

- |  |                                   |
|--|-----------------------------------|
| 17. $2m\left(-\frac{1}{3}n\right) + 3m(-2n)$ | 18. $1.2(3x + y) - 0.8(22x - 2y)$ |
|--|-----------------------------------|

19. **Standardized Test Practice** The velocity of an object  $t$  seconds after the object is dropped from the top of a tall building is about  $-9.8t$  meters per second (m/s). What is its velocity 2.5 seconds after it is dropped?

- A  $-24.5$  m/s      B  $-7.3$  m/s      C  $7.3$  m/s      D  $18.4$  m/s

# 2-4 Dividing Rational Numbers (Pages 84-87)

You can use the same rules of signs when dividing rational numbers that you used for multiplying.

<b>Dividing Two Rational Numbers</b>	The quotient of two numbers having the <i>same sign</i> is positive.
	The quotient of two numbers having <i>different signs</i> is negative.

If a fraction has one or more fractions in the numerator or denominator, it is a **complex fraction**. To simplify a complex fraction, rewrite it as a division expression.

**Examples**

a. Simplify  $\frac{\frac{4}{7}}{-8}$ .

Rewrite the complex fraction as  $\frac{4}{7} \div (-8)$ .

$$\begin{aligned} \frac{4}{7} \div (-8) &= \frac{4}{7} \cdot \left(-\frac{1}{8}\right) && \text{Multiply by } -\frac{1}{8}, \text{ the} \\ & && \text{reciprocal of } -8. \\ &= -\frac{4}{56} \text{ or } -\frac{1}{14} && \text{The signs are different,} \\ & && \text{so the product is} \\ & && \text{negative.} \end{aligned}$$

b. Simplify  $\frac{-2x + 10y}{5}$ .

$$\begin{aligned} \frac{-2x + 10y}{5} &= \frac{-2x}{5} + \frac{10y}{5} && \text{Divide each term by 5.} \\ &= -\frac{2}{5}x + 2y && \text{Simplify.} \end{aligned}$$

**Practice**

Simplify.

1.  $22 \div \left(\frac{11}{13}\right)$

2.  $24 \div \left(-\frac{1}{8}\right)$

3.  $\frac{-14}{-2}$

4.  $\frac{\frac{15}{-64}}{3}$

5.  $\frac{-\frac{30}{7}}{-10}$

6.  $\frac{8}{-\frac{4}{9}}$

7.  $\frac{-32m}{8}$

8.  $-18t \div \frac{8}{9}$

9.  $\frac{2a + 8}{4}$

10.  $\frac{8x + 42y}{6}$

11.  $\frac{-12h + (-18g)}{3}$

12.  $\frac{54s + 3w}{-6}$

Evaluate each expression if  $x = 4$ ,  $y = -5$ , and  $z = -1.5$ .

13.  $\frac{y}{z}$

14.  $\frac{xy}{xz}$

15.  $\frac{x + z}{3}$

16. **Standardized Test Practice** How many boxes of peanuts can you get from

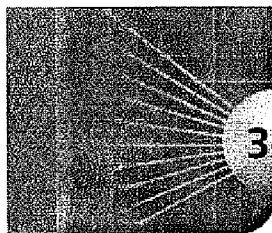
52 pounds of peanuts if each box holds  $1\frac{5}{8}$  pounds of peanuts?

A 84

B 32

C 26

D 50



# 3-5 Solving Equations with the Variable on Each Side (Pages 149–154)

To solve an equation that has the variable on both sides, use the properties of equality to write an equivalent equation that has the variable on only one side. Then solve. When you solve equations that contain grouping symbols, you may need to use the distributive property to remove the grouping symbols. Some equations may have no solution because there is no value of the variable that will result in a true equation. For example,  $x + 1 = x + 2$  has no solution; it cannot be true. An equation that is true for every value of the variable is called an **identity**. For example,  $x + x = 2x$  is true for every value of  $x$ .

### Examples

**a. Solve  $3(x - 2) = 4x + 5$ .**

First use the distributive property to remove the parentheses.

$$3x - 6 = 4x + 5$$

Next, collect all the terms with  $x$  on one side of the equal sign by subtracting  $3x$  from each side.

$$3x - 6 - 3x = 4x + 5 - 3x$$

$$-6 = x + 5$$

Add like terms.

$$-6 - 5 = x + 5 - 5$$

Subtract 5 from each side.

$$-11 = x$$

Simplify.

**b. Solve  $\frac{1}{2}y = \frac{1}{3}y + 2$ .**

First, multiply each side by 6, the LCD, to clear the fractions from the problem.

$$6 \cdot \frac{1}{2}y = 6\left(\frac{1}{3}y + 2\right)$$

$$6 \cdot \frac{1}{2}y = 6 \cdot \frac{1}{3}y + 6 \cdot 2$$

$$3y = 2y + 12$$

Next, collect all the terms with  $y$  on one side of the equal sign by subtracting  $2y$  from each side.

$$3y - 2y = 2y - 2y + 12$$

$$y = 12$$

### Try These Together

1. Solve  $4x + 3 = 5x + 7$ .

HINT: Subtract  $4x$  from each side.

2. Solve  $7 + 3t = \frac{6-t}{2}$ .

HINT: Multiply each side by 2.

### Practice

Solve each equation. Then check your solution.

3.  $18 + 2n = 4n - 9$

4.  $10 - 2.7y = y + 9$

5.  $\frac{2}{3}n + 6 = \frac{1}{4}n - 3$

6.  $11.1c - 2.4 = -8.3c + 6.4$

7.  $3 - 4x = 8x + 8$

8.  $\frac{3}{5}d + 5 = \frac{1}{3}d - 3$

9.  $3(2x - 1) = 9(x + 3)$

10.  $2(2x - 5) = 6x + 4$

11.  $-6(4x + 1) = 5 - 11x$

12.  $\frac{5}{6}(12p + 4) = -13p + 4$

13.  $-8\left(\frac{1}{4}n - 3\right) = n + 2$

14.  $\frac{2+t}{3} = 4 - \frac{6}{7}t$

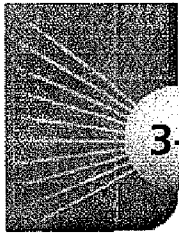
15. **Standardized Test Practice** Nine less than half  $n$  is equal to one plus the product of  $-\frac{1}{8}$  and  $n$ . Find the value of  $n$ .

A 24

B -21

C 8

D 16



# 3-4 Solving Multi-Step Equations (Pages 142-148)

<b>Solving Multi-Step Equations</b>	<ul style="list-style-type: none"> <li>• Work backward to isolate the variable and solve the equation.</li> <li>• Use subtraction to undo addition, and use addition to undo subtraction.</li> <li>• Use multiplication to undo division, and use division to undo multiplication.</li> </ul>
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**Consecutive integers** are integers in counting order, such as  $-3$ ,  $-2$ , and  $-1$ .

**Examples**

a. Solve  $\frac{2x - 3}{4} = 9$ .

Multiply each side by 4 to eliminate the fraction.

$$4\left(\frac{2x - 3}{4}\right) = 9(4)$$

$$2x - 3 = 36$$

Next, undo the subtraction by adding 3 to each side.

$$2x - 3 + 3 = 36 + 3$$

$$2x = 39$$

Last, undo the multiplication by dividing each side by 2.

$$\frac{2x}{2} = \frac{39}{2}$$

$$x = 19\frac{1}{2}$$

b. Find 3 consecutive odd integers whose sum is  $-3$ .

Let  $n$  = the least odd integer. Then  $n + 2$  = the next greater odd integer, and  $n + 4$  = the greatest of the three odd integers.

$$n + (n + 2) + (n + 4) = -3$$

$$3n + 6 = -3$$

Add like items.

$$3n + 6 - 6 = -3 - 6$$

Subtract 6 from each side.

$$3n = -9$$

Simplify.

$$\frac{3n}{3} = \frac{-9}{3}$$

Divide each side by 3.

$$n = -3$$

Simplify.

$n + 2 = -3 + 2$  or  $-1$  and  $n + 4 = -3 + 4$  or  $1$ , so the consecutive odd integers are  $-3$ ,  $-1$ , and  $1$ .

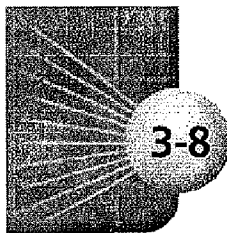
**Practice**

Solve each equation. Check your solution.

- |                             |                             |                              |                              |
|-----------------------------|-----------------------------|------------------------------|------------------------------|
| 1. $10 - 7p = -18$          | 2. $-1.9r + 9.3 = 15$       | 3. $6 = \frac{s}{3}$         | 4. $\frac{-4m - 3}{-6} = -9$ |
| 5. $-6 = \frac{-2n - 3}{4}$ | 6. $\frac{t}{5} - 4 = -10$  | 7. $11 = -7 - \frac{g}{3}$   | 8. $\frac{5}{6}b + 8 = -11$  |
| 9. $13 = -8 - 3t$           | 10. $-\frac{3 + n}{7} = -5$ | 11. $\frac{s + 4}{-2} = -16$ | 12. $3 - 9t = 21$            |

Define a variable, write an equation, and solve each problem.

13. Find two consecutive odd integers whose sum is 128.
14. Find three consecutive even integers whose sum is 90.
15. **Standardized Test Practice** Sally is eight years older than John. John is fourteen years older than Kareem. If the sum of all three ages is 90, how old is Kareem?
- A 8                      B 18                      C 28                      D 40



# 3-8 Solving Equations and Formulas

(Pages 166–170)

Some equations contain more than one variable. To solve an equation or formula for a specific variable, you need to get that variable by itself on one side of the equation. When you divide by a variable in an equation, remember that division by 0 is undefined.

When you use a formula, you may need to use **dimensional analysis**, which is the process of carrying units throughout a computation.

### Examples

- a. Solve the formula  $d = rt$  for  $t$ .**

The variable  $t$  has been multiplied by  $r$ , so divide each side by  $r$  to isolate  $t$ .

$$\frac{d}{r} = \frac{rt}{r} \text{ or } \frac{d}{r} = t$$

Thus  $t = \frac{d}{r}$ , where  $r \neq 0$ .

- b. Find the time it takes to drive 75 miles at an average rate of 35 miles per hour.**

Use the formula you found for  $t$  in Example A.

$$t = \frac{d}{r}$$

$$t = \frac{75 \text{ mi}}{35 \frac{\text{mi}}{\text{h}}}$$

Use dimensional analysis.

$$\frac{\text{mi}}{\text{mi}} = \frac{\text{mi}}{1} \cdot \frac{\text{h}}{\text{mi}} = \text{h}$$

$$t = 2\frac{1}{7} \text{ hours}$$

### Try These Together

1. Solve  $4a + b = 3a$  for  $a$ .

HINT: Begin by subtracting  $3a$  from each side.

2. Solve  $\frac{c+d}{3} = 2c$  for  $c$ .

HINT: Begin by multiplying each side by 3.

### Practice

Solve each equation for the variable specified.

3.  $f = epd$ , for  $e$

4.  $12g + 31h = -8g$ , for  $h$

5.  $y = mx + b$ , for  $b$

6.  $v = r + at$ , for  $r$

7.  $\frac{3x+y}{c} = 4$ , for  $c$

8.  $\frac{5xy+n}{2} = -6$ , for  $y$

9.  $m + n + 2p = 3$ , for  $m$

10.  $6y + z = bc - 2y$ , for  $y$

11.  $3x - 4y = 7$ , for  $y$

12.  $s = \frac{n}{2}(a + t)$ , for  $n$

13.  $v = \frac{4}{3}r$ , for  $r$

14.  $W = mgh$ , for  $g$

15.  $PV = nRT$ , for  $V$

16.  $G = F - D$ , for  $D$

17.  $6t + 62s = \frac{1}{2}(3t - 42s)$ , for  $t$

18.  $3c + 5d = 7d - 6c$ , for  $d$

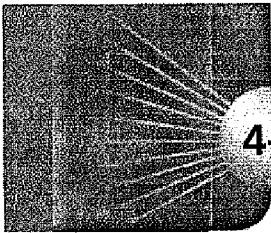
19. **Standardized Test Practice** Four ninths of a number  $c$  increased by 4 is 18 less than one eighth times another number  $d$ . Solve for  $c$ .

A  $c = \frac{9}{32}d + 31\frac{1}{2}$

B  $c = \frac{4}{72}d + \frac{4}{72}$

C  $c = \frac{9}{32}d - 49\frac{1}{2}$

D  $c = \frac{4}{72}d - 31\frac{1}{2}$



## 4-5

**Graphing Linear Equations** (Pages 218–223)

A **linear equation** may contain one or two variables with no variable having an exponent other than 1. A linear equation can be written in the form  $Ax + By = C$ , where  $A$ ,  $B$ , and  $C$  are any real numbers, and  $A$  and  $B$  are not both zero. To graph a linear equation, find at least two solutions of the equation. Then, plot the points and draw a straight line through them.

**Examples**

- a. Determine whether the equation  $y = 2x - 1$  is a linear equation. If it is, rewrite the equation in the form  $Ax + By = C$ .**

*This is a linear equation, since the equation contains only two variables and the power on each variable is 1. First, rewrite the equation so that both variables are on the same side of the equation.*

$$y = 2x - 1$$

$$-2x + y = -1 \quad \text{Subtract } 2x \text{ from each side.}$$

The equation is now in the form  $Ax + By = C$ , where  $A = -2$ ,  $B = 1$ , and  $C = -1$ .

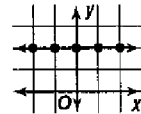
- b. Graph the equation  $y = 2$ .**

Select five values for the domain and make a table.

$x$	$y$	$(x, y)$
-2	2	$(-2, 2)$
-1	2	$(-1, 2)$
0	2	$(0, 2)$
1	2	$(1, 2)$
2	2	$(2, 2)$

*Note that because the equation does not contain the variable  $x$ ,  $x$  can be any value and the  $y$  value will still be 2.*

Then graph the ordered pairs and connect them to draw the line. Note that the graph of  $y = 2$  is a horizontal line through  $(0, 2)$ .

**Try These Together**

1. Rewrite the equation  $x = 3$  in the form  $Ax + By = C$ .

*HINT: Since there is no variable  $y$  in this equation, use the placeholder  $0y$ .*

2. Graph the equation  $3x - y = 5$ .

*HINT: To find values for  $y$  more easily, solve the equation for  $y$ . Subtract  $3x$  from each side and then divide each side by  $-1$ .*

**Practice**

Determine whether each equation is a linear equation. If an equation is linear, rewrite it in the form  $Ax + By = C$ .

3.  $y = 2x^2 - 3$

4.  $x = 2y + 8$

5.  $y = -1$

6.  $y = -4x + 1$

7.  $3x = 5y + 7$

8.  $8 - y = x$

Graph each equation.

9.  $y = x + 4$

10.  $y = 3x - 1$

11.  $y = 3 - 2x$

12.  $y - 3 = 0$

13.  $y + 5 = 0$

14.  $x - 2 = 0$

15.  $x - y = 6$

16.  $x + y = 15$

17.  $2x + y = 4$

18. **Standardized Test Practice** Write the equation  $y = 2x - 8$  in the standard form  $Ax + By = C$ .

A  $y + 2x = -8$

B  $y - 2x = -8$

C  $-2x + y = -8$

D  $2x + y = -8$

